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HEAT TRANSFER IN THE THERMAL ENTRY LENGTH WITH LAMINAR FLOW BETWEEN PARALLEL WALLS AT UNEQUAL TEMPERATURES

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Abstract—To extend the well-known solution of laminar flow in a flat duct to the case of unequal wall temperatures requires the determination of an additional set of eigenvalues and functions, the first eight of which were determined by computer.

The Nusselt numbers do not become constant, in this case, until a linear temperature gradient is established in the fluid, and this requires very long entry lengths. The entry lengths also depend on the magnitude of the entry temperature in relation to the values of the wall temperatures.

NOMENCLATURE

a,	coefficient in series expansion;
A, B, C,	constants defined in text;
K,	thermal conductivity of fluid;
k,	wall temperature constant defined in text;
Nu,	Nusselt number;
Pr,	Prandtl number;
q,	Heat-transfer rate;
Re,	Reynolds number, $4u_m y'_a/\nu$;
t,	fluid temperature;
u,	fluid velocity;
X,	function of x;
x,	normalized co-ordinate in flow direction;
x', y', z',	cartesian co-ordinates;
y,	normalized co-ordinate perpendicular to wall;
Y,	function of y.

Greek symbols

α,	thermal diffusivity of fluid;
λ,	eigenvalue;
θ,	normalized temperature;
ν,	kinematic viscosity.

Subscripts

a,	value at one wall;
b,	value at other wall;
E,	even function of y;
fd,	fully developed temperature distribution;
m,	mean value;
n,	eigenvalue number;
O,	odd function of y;
WM,	arithmetic mean of wall temperatures.

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INTRODUCTION

THE problem is treated as one of steady-state heat transfer to or from a one-dimensional flow of fluid between infinite parallel flat walls in laminar flow with an established velocity profile. Consider cartesian co-ordinates x', y' and z' (see Fig. 1); x' is the centre-line co-ordinate in the flow direction, and y' and z' lie perpendicular and parallel to the walls, respectively. The equation for steady heat conduction in the fluid with negligible dissipative effects may be written:

$$u\,\frac{\partial t}{\partial x'} = \alpha\left[\frac{\partial^2 t}{\partial x'^2} + \frac{\partial^2 t}{\partial y'^2} + \frac{\partial^2 t}{\partial z'^2}\right]. \qquad (1)$$

In this paper, two-dimensional heat transfer in the xy-plane is considered, and all derivations with respect to z are taken as zero. Heat conduction in the flow direction is assumed to be negligible, and consequently $\partial^2 t/\partial x'^2$ is neglected by comparison with $\partial^2 t/\partial y'^2$.

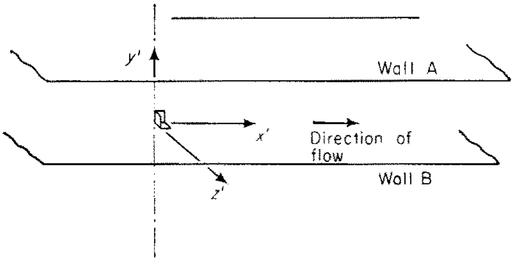

FIG. 1. Co-ordinate system.

Equation (1) now becomes

$$u \frac{\partial t}{\partial x'} = \alpha \frac{\partial^2 t}{\partial y'^2}. \tag{2}$$

If uniform fluid properties are assumed, the velocity profile may be expressed as:

$$u = u_m \frac{3}{2} \left[1 - \left(\frac{y'}{y'_a} \right)^2 \right] \tag{3}$$

where $y'_a = -y'_b$ is the distance from mid-stream to the wall. Now if we put $y = y'/y'_a$ and $x = x'/Re \cdot Pr \cdot y'_a$, equation (2) may be re-written for uniform fluid properties as:

$$\frac{3}{8}(1 - y^2) \frac{\partial t}{\partial x} = \frac{\partial^2 t}{\partial y^2}. \tag{4}$$

Equation (4) together with prescribed boundary conditions may be used to determine the temperature distribution in the fluid. Graetz [1, 2] published a solution of the corresponding equation for laminar flow in a straight circular tube, which involved separation of variables in x and y resulting in an eigenvalue problem. This method has since been applied to laminar flow between parallel walls by several authors [3–6]. However, the interesting case of heat transfer in the thermal entry region for laminar flow between parallel walls at uniform but unequal temperatures does not appear to have been investigated. It can be regarded as the limiting case of heat transfer in an annulus with uniform unequal wall temperatures and radius ratio of unity. It will be seen that such a problem involves two distinct sets of eigenvalues related to odd and even sets of eigenfunctions, respectively.

BOUNDARY CONDITIONS AND NORMALIZED TEMPERATURE FUNCTIONS

The boundary conditions for equation (4) are:

(i) $t = t_0 \qquad -1 \leqslant y \leqslant +1 \qquad -\infty \leqslant x < 0$

(ii) $t = t_a \qquad y = +1 \qquad 0 \leqslant x \leqslant +\infty$

(iii) $t = t_b \qquad y = -1 \qquad 0 \leqslant x \leqslant +\infty$

(iv) $t = t_{fd}(y) \qquad -1 \leqslant y \leqslant +1 \qquad x = +\infty$

where t_0 is the initial fluid temperature and $t_{fd}(y)$ is the fully developed temperature profile (see Fig. 2). Introducing the normalized temperature,

$$\theta = \frac{t - t_{WM}}{t_a - t_{WM}}$$

where $t_{WM} = (t_a + t_b)/2$, θ may be expressed as the sum of a fully developed temperature distribution which is a function of y alone and an entry temperature distribution which is a function of x and y

$$\theta = \theta_1 + \theta_2 \tag{5}$$

where

$$\theta_1 = \frac{t_{fd} - t_{WM}}{t_a - t_{WM}}$$

and

$$\theta_2 = \frac{t - t_{fd}}{t_a - t_{WM}}$$

since θ_1 is a function of y alone and equation (4) is linear in t, it is possible to write:

$$\frac{d^2\theta_1}{dy^2} = 0. \tag{6}$$

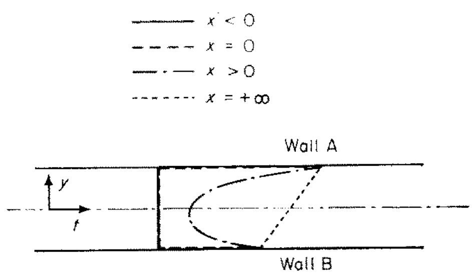

FIG. 2. Typical temperature profiles.

The solution of equation (6) is $\theta_1 = Ay + B$, the boundary conditions are $\theta_1 = 1$ when $y = 1$, and $\theta_1 = -1$ when $y = -1$, giving

$$\theta_1 = y. \tag{7}$$

Defining a constant,

$$k = \frac{t_0 - t_{WM}}{t_a - t_{WM}}$$

the boundary conditions for θ and θ_2 become:

(i) $\theta = k \quad \theta_2 = k - y \quad -1 \leqslant y \leqslant +1 \quad -\infty \leqslant x < 0$

(ii) $\theta = k \quad \theta_2 = k - y \quad -1 < y < +1 \quad x = 0$

(iii) $\theta = 1 \quad \theta_2 = 0 \quad y = +1 \quad 0 \leqslant x \leqslant +\infty$

(iv) $\theta = -1 \quad \theta_2 = 0 \quad y = -1 \quad 0 \leqslant x \leqslant +\infty$

(v) $\theta = y \quad \theta_2 = 0 \quad -1 \leqslant y \leqslant +1 \quad x = +\infty.$

From equation (4), θ_2 may be expressed as

$$\frac{3}{8}(1 - y^2)\frac{\partial \theta_2}{\partial x} = \frac{\partial^2 \theta_2}{\partial y^2}. \tag{8}$$

SOLUTION

To separate the variables in x and y for equation (8), we put

$$\theta_2 = XY$$

where X is a function of x only and Y a function of y only; this gives:

$$\frac{3}{8}\frac{1}{X}\frac{\mathrm{d}X}{\mathrm{d}x} = \frac{1}{(1 - y^2)Y}\frac{\mathrm{d}^2 Y}{\mathrm{d}y^2} = -\lambda^2.$$

The solution for X with boundary condition (v) for θ_2 is

$$X = \exp(-8\lambda^2 x/3)$$

and the equation for Y is

$$\frac{\mathrm{d}^2 Y}{\mathrm{d}y^2} + \lambda^2(1 - y^2)Y = 0. \tag{9}$$

Equation (9), together with the boundary conditions for θ_2, constitute a Sturm–Liouville system which may be solved for discrete values of λ called characteristic values or eigenvalues. The resulting functions of y (eigenfunctions) may be even or odd in y, and two distinct sets of solutions may therefore be noted:

(1) Even eigenfunctions $Y_{E1}, Y_{E2} \ldots Y_{En}$ and the corresponding eigenvalues $\lambda_{E1}, \lambda_{E2} \ldots \lambda_{En}$.

(2) Odd eigenfunctions $Y_{O1}, Y_{O2} \ldots Y_{On}$ and the corresponding eigenvalues $\lambda_{O1}, \lambda_{O2} \ldots \lambda_{On}$.

The solution for θ_2 is:

$$\theta_2 = \sum_{n=1}^{\infty} C_{En} Y_{En} \exp(-8\lambda_{En}^2 x/3) + \sum_{n=1}^{\infty} C_{On} Y_{On} \exp(-8\lambda_{On}^2 x/3) \tag{10}$$

where C_{En} and C_{On} are constants to be determined from the boundary condition (ii) for θ_2. From the properties of the Sturm–Liouville system,

$$C_{En} = \frac{\int_{-1}^{1} \theta_2(x = 0)(1 - y^2) Y_{En}\,\mathrm{d}y}{\int_{-1}^{1}(1 - y^2) Y_{En}^2\,\mathrm{d}y} = \frac{k\int_{-1}^{1}(1 - y^2) Y_{En}\,\mathrm{d}y}{\int_{-1}^{1}(1 - y^2) Y_{En}^2\,\mathrm{d}y} - \frac{\int_{-1}^{1} y(1 - y^2) Y_{En}\,\mathrm{d}y}{\int_{-1}^{1}(1 - y^2) Y_{En}^2\,\mathrm{d}y} = \frac{k\int_{0}^{1}(1 - y^2) Y_{En}\,\mathrm{d}y}{\int_{0}^{1}(1 - y^2) Y_{En}^2\,\mathrm{d}y} \tag{11}$$

since the product yY_{En} is an odd function of y; also

$$C_{On} = \frac{\int_{-1}^{1} \theta_2(x = 0)(1 - y^2) Y_{On}\,\mathrm{d}y}{\int_{-1}^{1}(1 - y^2) Y_{On}^2\,\mathrm{d}y} = -\frac{\int_{0}^{1} y(1 - y^2) Y_{On}\,\mathrm{d}y}{\int_{0}^{1}(1 - y^2) Y_{On}^2\,\mathrm{d}y} \tag{12}$$

since Y_{On} is an odd function of y.

Integrating equation (9) from 0 to 1 for an even eigenfunction gives:

$$\int_0^1 (1 - y^2) Y_{En}\,\mathrm{d}y = -\frac{1}{\lambda^2}\left[\frac{\mathrm{d}Y_{En}}{\mathrm{d}y}\right]_{y=1}. \tag{13}$$

Multiplying equation (9) by y and integrating from 0 to 1 for an odd eigenfunction gives:

$$\int_0^1 y(1 - y^2) Y_{On}\,\mathrm{d}y = -\frac{1}{\lambda^2}\left[\frac{\mathrm{d}Y_{On}}{\mathrm{d}y}\right]_{y=1}. \tag{14}$$

Graetz [2] showed that alternative expressions for the denominations of (11) and (12) may be obtained. From equation (9) it follows that, for any solution Y,

$$\left[\frac{\partial Y}{\partial \lambda}\cdot\frac{\partial Y}{\partial y} - Y\frac{\partial}{\partial y}\left(\frac{\partial Y}{\partial \lambda}\right)\right]_{y=1} = 2\lambda \int_0^1 (1-y^2)Y^2\,\mathrm{d}y.$$

If Y be an eigenfunction then $Y = 0$, $y = 1$ and

$$\int_0^1 (1-y^2)Y_n^2\,\mathrm{d}y = \frac{1}{2\lambda}\left[\frac{\partial Y_n}{\partial \lambda}\cdot\frac{\partial Y_n}{\partial y}\right]_{y=1} \tag{15}$$

It follows from (11), (13) and (15) that

$$C_{En} = -\frac{2k}{\lambda_{En}(\partial Y_{En}/\partial \lambda_{En})_{y=1}} \tag{16}$$

and from (12), (14) and (15) that

$$C_{On} = \frac{2}{\lambda_{On}(\partial Y_{On}/\partial \lambda_{On})_{y=1}}. \tag{17}$$

NUSSELT NUMBERS

Expressions for the Nusselt numbers at each wall may now be derived. If a sign convention is adopted to give positive heat-transfer rates and Nusselt numbers at both walls for $x = \infty$, then the heat-transfer rate from wall a to the stream is given by

$$q_a = K\left(\frac{\partial t}{\partial y'}\right)_{y'=y'_a}$$

and from the stream to wall b is given by

$$q_b = K\left(\frac{\partial t}{\partial y'}\right)_{y'=+y'_b}.$$

The Nusselt number for wall a may be written

$$Nu_a = \frac{q_a}{t_a - t_m}\cdot\frac{4y'_a}{K}$$

where t_m is the bulk mean temperature of the fluid;

$$t_m = \frac{\int_{-1}^{+1} tu\,\mathrm{d}y}{\int_{-1}^{+1} u\,\mathrm{d}y}.$$

Alternatively

$$Nu_a = \frac{4}{1-\theta_m}\left(\frac{\mathrm{d}\theta}{\mathrm{d}y}\right)_{y=1} \tag{18}$$

where

$$\theta_m = \frac{\int_{-1}^{+1} \theta u\,\mathrm{d}y}{\int_{-1}^{+1} u\,\mathrm{d}y}. \tag{19}$$

In the same manner, the Nusselt number at wall b may be written:

$$Nu_b = \frac{q_b}{t_m - t_b}\cdot\frac{4y'_a}{K}$$

hence

$$Nu_b = \frac{4}{1+\theta_m}\left(\frac{\mathrm{d}\theta}{\mathrm{d}y}\right)_{y=-1}. \tag{20}$$

From equations (3) and (19) the normalized bulk mean temperature may be written

$$\theta_m = \frac{3}{4}\int_{-1}^{+1} \theta(1-y^2)\,\mathrm{d}y.$$

Substituting for θ by means of equations (5), (7), (10), (13) and (16) gives the final expression for the normalized bulk mean temperature:

$$\theta_m = 3k\sum_{n=1}^{\infty}\frac{1}{\lambda_{En}^3}\frac{(\partial Y_{En}/\partial y)_{y=1}}{(\partial Y_{En}/\partial \lambda_{En})_{y=1}}\exp(-8\lambda_{En}^2 x/3) \tag{21}$$

since integrals of odd functions from $y = +1$ to $y = -1$ are zero. Final expressions for the Nusselt numbers may now be written, with the aid of equations (5), (7), (10), (16–18), (20) and (21), as:

$$Nu_a = \frac{1 - 2k\sum_{n=1}^{\infty}\frac{1}{\lambda_{En}}\frac{(\partial Y_{En}/\partial y)_{y=1}}{(\partial Y_{En}/\partial \lambda_{En})_{y=1}}\exp(-8\lambda_{En}^2 x/3) + 2\sum_{n=1}^{\infty}\frac{1}{\lambda_{On}}\frac{(\partial Y_{On}/\partial y)_{y=1}}{(\partial Y_{On}/\partial \lambda_{On})_{y=1}}\exp(-8\lambda_{On}^2 x/3)}{\frac{1}{4} - \frac{3k}{4}\sum_{n=1}^{\infty}\frac{1}{\lambda_{En}^3}\frac{(\partial Y_{En}/\partial y)_{y=1}}{(\partial Y_{En}/\partial Y_{En})_{y=1}}\exp(-8\lambda_{En}^2 x/3)} \tag{22}$$

$$Nu_b = \frac{1 + 2k \sum_{n=1}^{\infty} \frac{1}{\lambda_{En}} \frac{(\partial Y_{En}/\partial y)_{y=1}}{(\partial Y_{En}/\partial \lambda_{En})_{y=1}} \exp(-8\lambda_{En}^2 x/3) + 2 \sum_{n=1}^{\infty} \frac{1}{\lambda_{On}} \frac{(\partial Y_{On}/\partial y)_{y=1}}{(\partial Y_{On}/\partial y)_{y=1}} \exp(-8\lambda_{On}^2 x/3)}{\frac{1}{4} + \frac{3k}{4} \sum_{n=1}^{\infty} \frac{1}{\lambda_{En}^3} \frac{(\partial Y_{En}/\partial y)_{y=1}}{(\partial Y_{En}/\partial \lambda_{En})_{y=1}} \exp(-8\lambda_{En}^2 x/3)} \quad (23)$$

since

$$\left(\frac{\partial Y_{En}}{\partial y}\right)_{y=1} = -\left(\frac{\partial Y_{En}}{\partial y}\right)_{y=-1}$$

and

$$\left(\frac{\partial Y_{On}}{\partial y}\right)_{y=1} = \left(\frac{\partial Y_{On}}{\partial y}\right)_{y=-1}$$

CALCULATION

From the foregoing it can be seen that the solution requires computation of

$$\lambda_{En},\quad \lambda_{On},\quad \left(\frac{\partial Y_{En}}{\partial y}\right)_{y=1},\quad \left(\frac{\partial Y_{On}}{\partial y}\right)_{y=1},\quad \left(\frac{\partial Y_{En}}{\partial \lambda}\right)_{y=1},\quad \left(\frac{\partial Y_{On}}{\partial \lambda}\right)_{y=1}.$$

Graetz [1] expressed the eigenfunctions for the round tube as infinite power series. The eigenvalues appeared as the roots of a given series. Successive terms in the series were of different sign and the calculation of the roots became very laborious after the first three. Prins *et al.* [5] give the first three even eigenvalues for the flat-duct case and the related derivatives. Schenk and Dumore [6] present the first five values. Sellars *et al.* [7] extended the solution to the case of uniform heat input and gave approximations to all the eigenvalues. Brown [3] applied an electronic computer to both the round-tube and flat-duct case and calculated by the series method to a high degree of accuracy using fifty significant figures. He obtained the first ten eigenvalues and derivatives.

The present authors carried out their computation on the Manchester Mercury Computer, and in view of the extreme precision necessary in the above method it was considered simpler to integrate equation (9) from $y = 0$ to 1 on a step-by-step basis for trial values of λ with the Runge–Kutta method. Two hundred equal steps were taken with variables:

$$y_1 = \frac{dY}{dy};\ y_2 = Y;\ y_3 = \frac{d}{dy}\left(\frac{dY}{d\lambda}\right);\ y_4 = \frac{dY}{d\lambda}.$$

The corresponding equations for integration are:

$$\frac{dy_1}{dy} = -\lambda^2(1 - y^2)y^2$$

$$\frac{dy_2}{dy} = y_1$$

$$\frac{dy_3}{dy} = -\lambda^2(1 - y^2)y_4 - 2\lambda(1 - y^2)y_2$$

$$\frac{dy_4}{dy} = y_3$$

and the boundary conditions at the starting point $y = 0$ are:

for $Y = Y_E$: $y_1 = 0,\ y_2 = 1,\ y_3 = 0,\ y_4 = 0$;

for $Y = Y_O$: $y_1 = 1,\ y_2 = 0,\ y_3 = 0,\ y_4 = 0$.

RESULTS

The first eight values of

$$\lambda_{En}, \lambda_{On}, \left(\frac{dY_{En}}{d\lambda}\right)_{y=1},\quad \left(\frac{dY_{On}}{d\lambda}\right)_{y=1},\quad \left(\frac{dY_{En}}{dy}\right)_{y=1} \text{ and } \left(\frac{dY_{On}}{\partial y}\right)_{y=1}$$

are given in Table 1 for Y_E and $Y_O < 1 \times 10^{-8}$, respectively, at $y = 1$. The even eigenvalues are compared with those reported by Brown [3] in Table 2. Curves of Nu_a and Nu_b against x are presented for values of k of 0, 1, 2 and 3 in Fig. 3.

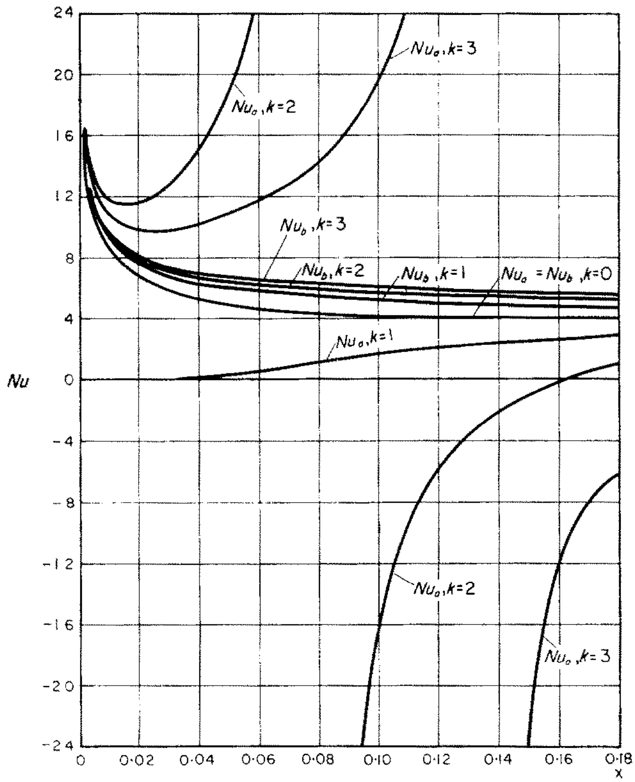

FIG. 3. Variation of Nusselt Number along duct.

Table 1. Eigenvalues and derivatives

n	λ_{En}	$\left(\frac{\partial Y_{En}}{\partial y}\right)_{y=1}$	$\left(\frac{\partial Y_{En}}{\partial \lambda}\right)_{y=1}$
1	1·681595	− 1·429156	− 0·990437
2	5·669858	3·807069	1·179107
3	9·668243	− 5·920236	− 1·286249
4	13·66766	7·892533	1·362019
5	17·66738	− 9·770940	− 1·421324
6	21·66722	11·57980	1·470396
7	25·66714	− 13·33385	− 1·512447
8	29·66710	15·04291	1·549358
n	λ_{On}	$\left(\frac{\partial Y_{On}}{\partial y}\right)_{y=1}$	$\left(\frac{\partial Y_{On}}{\partial \lambda}\right)_{y=1}$
1	3·672290	− 0·7144592	− 0·2951278
2	7·668809	0·6345106	0·1608078
3	11·66790	− 0·5920333	− 0·1135084
4	15·66750	0·5637611	0·08883561
5	19·66729	− 0·5428368	− 0·07352136
6	23·66717	0·5263594	0·06301967
7	27·66711	− 0·5128434	− 0·05533502
8	31·66710	0·5014311	0·04944843

Table 2. Even eigenvalues calculated by Brown [3]

n	λ_{En}
1	1·6815953222
2	5·6698573459
3	9·6682424625
4	13·6676614426
5	17·6673735653
6	21·6672053243
7	25·6670964863
8	29·6670210447

CONCLUSIONS

Table 1 shows that the odd eigenvalues take values which are approximately half-way between those of the even eigenvalues. The even eigenvalues approximate closely to those reported by Brown [3].

Sellars *et al.* [7] gave an expression for the asymptotic, even eigenvalues. A slight modification to their work shows that the eigenvalues, both even and odd, are approximately given by

$$\lambda_j = 2_j + 5/3$$

where j is the number of times the function vanishes in the interval between $y = -1$ and $+1$. Thus, for the even eigenvalues j is 0, 2, 4, etc., and, for the odd, j is 1, 3, 5, etc.

The heat transfer in the entry length is more complicated than in the corresponding case with equal wall temperatures. In the latter, the transverse temperature profile is the same shape for all values of x which are sufficiently large for the influence of the second and higher even eigenfunctions to be negligible. This gives a Nusselt number which is uniform along the length of the duct after the development length, that is after the influence of the second even eigenfunction has died away. The present study indicates that this does not occur with unequal wall temperatures. The final condition at $x = +\infty$ for this case is given by the simple temperature distribution, $\theta = y$ with $Nu_a = Nu_b = 4$, and it is necessary for the influence of all even and odd eigenfunctions to die away before this is reached. Very long lengths of duct are therefore necessary for an established temperature distribution and uniform Nusselt numbers to occur.

For the special case of $k = 0$, only odd eigenfunctions are involved and $Nu_a = Nu_b$ for all x. The length of duct necessary to obtain an established temperature profile is shorter than for the other values of k studied.

Infinities and zeroes occur when $x > 0$ in Nu_a if $1 < k < \infty$ and in Nu_b if $-\infty < k < -1$. The Nusselt numbers become infinite when the bulk mean temperature of the fluid is equal to the appropriate wall temperature. Zero Nusselt numbers indicate a change in the direction of the heat transfer at the corresponding wall. Fig. 3 shows that the values of x at the infinite and zero Nusselt numbers, respectively, approach $x = 0$ as k tends to unity for Nu_a and as k tends to -1 for Nu_b.

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Résumé—Pour étendre la solution de l'écoulement laminaire dans un conduit plat au cas où les températures de paroi sont différentes, il faut déterminer un système supplémentaire de 8 valeurs et fonctions; les 8 premières ont été calculées à l'aide d'une machine électronique.

Dans ce cas, les nombres de Nusselt ne deviennent constant que lorsqu'un gradient de température linéaire s'est établi dans le fluide, ceci nécessite des longueurs d'entrée très grandes. Les longueurs d'entrée dépendent également de l'importance de la température d'entrée par rapport aux températures de paroi.

Zusammenfassung—Um die bekannte Lösung für Laminarströmung in ebenen Kanälen auf den Fall ungleicher Wandtemperaturen auszudehnen, ist eine Reihe von Eigenwerten und Funktionen zu bestimmen. Die ersten acht davon wurden auf einer Rechenmaschine ermittelt.

Die Nusseltzahl wird in diesem Fall erst dann konstant, wenn sich in der Flüssigkeit ein linearer Temperaturgradient eingestellt hat; dieser Umstand erfordert sehr lange Einlauflängen. Darüber hinaus hängen die Einlauflängen noch ab vom Verhältnis der Eintrittstemperatur zur Wandtemperatur.

Аннотация—Чтобы развить имеющиеся в литературе решения задачи о ламинарном течении в плоском канале на случай, когда температуры стенок канала неодинаковы, требуется определить дополнительную совокупность собственных значений и функций, первые восемь из которых были определены на счётно-решающем устройстве.

В этом случае величины числа Нуссельта будут переменными до тех пор пока не установится линейный температурный градиент. Для установления такого температурного поля требуется большая длина входного участка, протяженность которого зависит также от отношения величины температуры на входе к величине температуры стенок.